Solutions for Physics 9C-A MT #2 (2016)

1-(a) Since $V_{10} = 120V = Q_{10}/C_{10}$,

$$Q_{10} = C_{10}V_{10} = 960\,\mu C.$$

Since $V_{20} = 0 = Q_{20}/C_{20}$

$$Q_{20} = 0.$$

1-(b) From charge conservation, $Q_1 + Q_2 = Q_{10}$. The potential drop $V_1 = V_2$, and is related to $Q_1 + Q_2 = Q_{10}$ through the network capacitance $C_{network} = C_{10} + C_{20}$ as

$$V_1 = V_2 = \frac{Q_1 + Q_2}{C_{network}} = \frac{Q_{10}}{C_{10} + C_{20}}$$

$$= \frac{960\,\mu C}{8\,\mu F + 4\,\mu F} = 80V.$$

Thus

$$Q_1 = C_{10} \cdot V_1 = 640\,\mu C$$

$$Q_2 = C_{20} \cdot V_1 = 320\,\mu C.$$

1-(c) Again the charge conservation requires

$$Q_1 + Q_2 = Q_{10} = 960\,\mu C$$

But now $C_2 = k C_{20} = 8\,\mu C$. As a result the network capacitance

$$C_{network} = C_{10} + C_2 = 16\,\mu C$$

Thus $V_1 = V_2 = \dfrac{Q_1 + Q_2}{C_{network}} = \dfrac{960\,\mu C}{16\,\mu C} = 60V$

Sanity check:

$Q_1 = C_{10} \cdot V_1 = 480\,\mu C$

$Q_2 = C_2 \cdot V_2 = 480\,\mu C$

2-(a) C_1 and C_2 are parallel, thus

$$C_{ac} = C_{12} = C_1 + C_2 = 20\,\mu F.$$

C_{12} and C_3 are in series, thus

$$C_{ab} = C_{123} = \frac{C_{12} \cdot C_3}{C_{12} + C_3} = 4\,\mu F$$

※

2-(b) Since $V_{ab} = 40V$,

$$Q_3 = (Q_1 + Q_2) = C_{123} \cdot V_{ab} = 160\,\mu C.$$

Then the potential drop across a to c

$$V_{ac} = (Q_1 + Q_2)/C_{ac} = Q_3/C_{ac} = 8V$$

$$\therefore \; Q_1 = V_{ac} \cdot C_1 = 88\,\mu C$$

$$Q_2 = V_{ac} \cdot C_2 = 72\,\mu C$$ ※

Alternatively,

$$V_{ac} = V_{ab} - V_{cb} = V_{ab} - Q_3/C_3 = 40V - 32V$$

$$= 8V$$

$$\therefore Q_1 = V_{ac} \cdot C_1 = 88\,\mu C; \; Q_2 = V_{ac} \cdot C_2 = 72\,\mu C$$ ※

3-(a) $R_{12} = R_1 + R_2 = 12\Omega$

$$R_{123} = \frac{R_{12} \cdot R_3}{R_{12} + R_3} = \frac{12\Omega \cdot 24\Omega}{36\Omega} = 8\Omega$$

$$R_{1234} = R_{123} + R_4 = 24\Omega = R_{ab}$$

3-(b) $P_{ab} = V_{ab}^2 / R_{ab} = \mathcal{E}^2 / R_{ab} = 216\,W.$

3-(c) $I_4 = V_{ab} / R_{ab} = 3A$

$P_4 = I_4^2 R_4 = 144\,W$

$V_3 = V_1 + V_2 = I_4 \cdot R_{123} = 24V$

$\therefore P_3 = V_3^2 / R_3 = 24W$

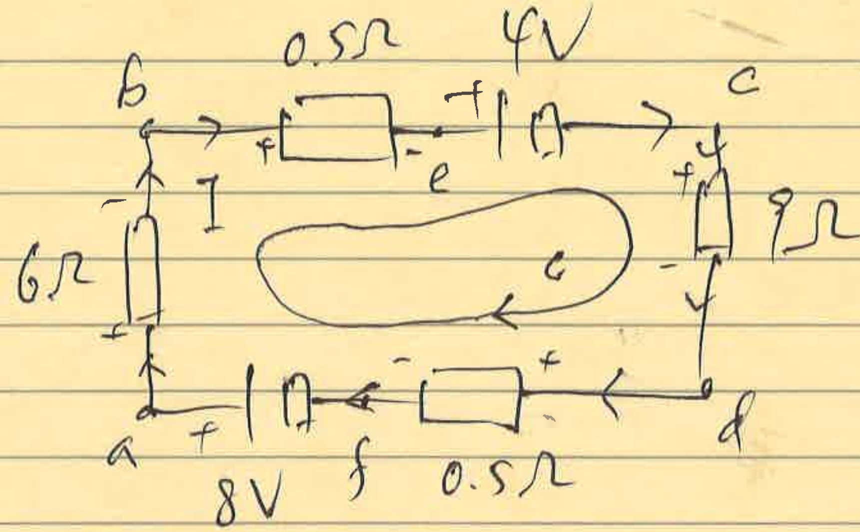
$I_{12} = (V_1 + V_2)/R_{12} = V_3 / R_{12} = 2A$

$\therefore P_1 = I_{12}^2 R_1 = 12W$

$P_2 = I_{12}^2 R_2 = 36W$

$P_1 + P_2 + P_3 + P_4 = 216\,W$, same as in Part 3(b)

4-(a)



Assign I as indicated. Along a clockwise loop,

$$\oint_{c} \vec{E}\cdot d\vec{l} = \int_{abecdfa} \vec{E}\cdot d\vec{l}$$

$$= 6I + 0.5I + 4 + 9I + 0.5I - 8$$

$$= 0$$

$$\therefore\ 16I = 4$$

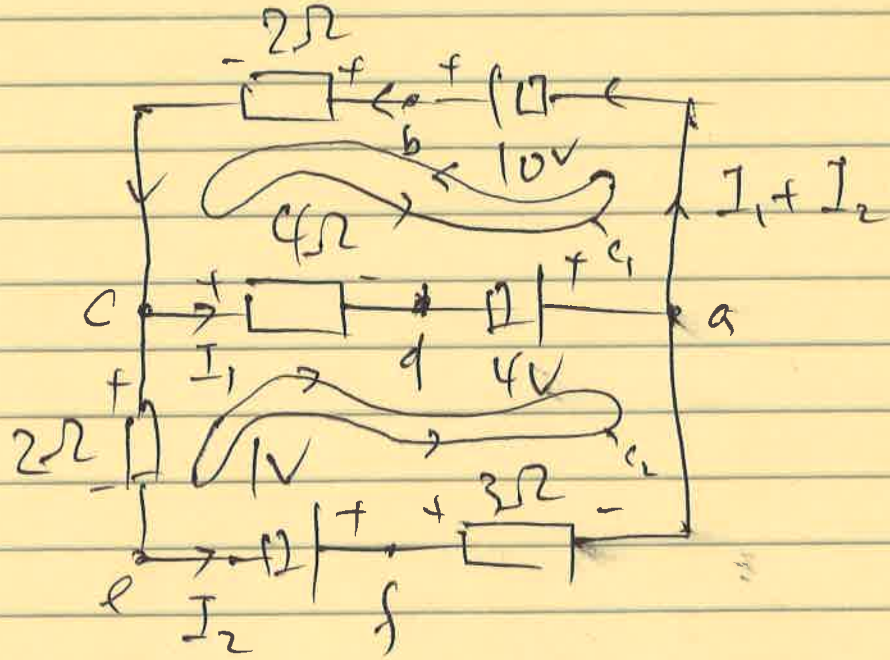
$I = 0.25\,A$ in the direction as assigned.

4-(b) $V_{ac} = V_a - V_c = V_{ab} + V_{be} + V_{ec} = (6 + 0.5)I + 4$

$$= 5\tfrac{5}{8}\,V = 5.625\,V$$

4-(c) $V_{bd} = V_{be} + V_{ec} + V_{cd} = (9.5)\cdot I + 4 = 6\tfrac{3}{8}\,V = 6.375\,V$

5-(a)



Assign two currents I_1 and I_2 as indicated.
Assign two clockwise loops c_1 and c_2 as indicated.

Along c_1:

$$\oint_{c_1} \vec{E}\cdot d\vec{l} = 0 = \int_{abcda} \vec{E}\cdot d\vec{l}$$

$$= -10 + (I_1+I_2)\cdot 2 + 4I_1 - 4 = 0$$

$$\therefore\ 6I_1 + 2I_2 = 14 \quad \text{or} \quad 3I_1 + I_2 = 7 \cdots ①$$

Along c_2:

$$\oint_{c_2} \vec{E}\cdot d\vec{l} = 0 = \int_{adcefa} \vec{E}\cdot d\vec{l}$$

$$\oint_{adeefa} \vec{E}\cdot d\vec{l} = 0 = 4 - 4I_1 + 2I_2 - 1 + 3I_2 = 0$$

$$\therefore \quad 4I_1 - 5I_2 = 3 \quad \cdots\cdots \quad (2)$$

(1) $\times$ 5 + (2):

$$19 I_1 = 38 \quad \cdots\cdots \quad (3)$$

$\therefore$ $I_1 = 2$ A as assigned in direction

From Eq. (1):

$I_2 = 7 - 3I_1 = 1$A as assigned in direction

$\therefore$ $I_1 = 2$A (through 4Ω)

$I_2 = 1$A (through 2Ω and 3Ω)

$I_1 + I_2 = 3$A (through 2Ω at the top)

5-(b) $V_{df} = V_{da} + V_{af} = -4V - 3I_2 = -7V$